4

Proportional Reasoning and Statistics

CHAPTER 4 Proportional Reasoning and Statistics

How often do you need to compare one thing to another? Perhaps you compare prices when you are buying something or you compare heights of the basketball players on the court. In mathematics, comparing one thing to another is an important **strategy** to learn about how they are related. We do it when dealing with ratios, fractions, decimals, percents, and similar figures.

You will begin comparing parts and wholes in Section 4.1. You will use these relationships to find percentages, something that you use in your daily life.

In Section 4.2, you will return to the Calaveras Frog Jumping Contest data and use new analysis tools to compare groups of frogs. You will also look at ways to display and interpret data.

Guiding Questions

Think about these questions throughout this chapter:

What's the relationship?

How can I represent the data?

What is the part?

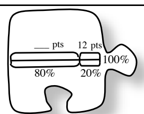
What is the whole?

Section 4.3 will have you investigate how shapes change using mathematics you already know. You will also determine the unknown length of a side in a figure when given information about the lengths of other sides in the figure and in related figures.

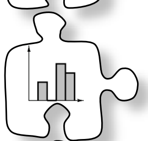
In this chapter, you will learn how to:

- Find and use percentages to solve problems.
- Use measures of central tendency, histograms, stem-and-leaf plots, and box-and-whisker plots to compare data.
- Compare shapes and use similarity to find missing side lengths of polygons, especially triangles.

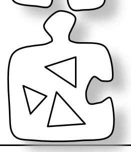
Chapter Outline



Section 4.1 This section introduces a linear diagram that you will use to represent relationships between parts and the whole to solve problems.

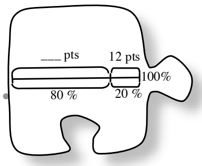


Section 4.2 This section introduces you to multiple ways of representing data, including histograms, stem-and-leaf plots, and box-and-whisker plots. You will compare sets of data using these representations.



Section 4.3 This section will introduce similarity and congruence for polygons.

4.1.1 How can I find a percentage?



Part-Whole Relationships

Food labels tell you about what is in the food you eat. The nutrition facts label lists the percentages of vitamins and minerals in each serving. But often it does not tell you exactly how much of each vitamin or mineral is in the food or how much you should have each day to be healthy.

If you know how much Vitamin C is in a serving, can you figure out how much is needed in a day? To help you answer questions like that, this section will develop **strategies** that will help you find information about parts and wholes.

Nutrition Facts

Serving Size 1 cup (228g)
Serving Per Container 4

Amount Per Serving	
Calories 250	Calories from Fat 110
	% Daily Value*
Total Fat 12g	**18%**
Saturated Fat 3g	**15%**
Cholesterol 30mg	**10%**
Sodium 470mg	**20%**
Total Carbohydrate 31g	**10%**
Dietary Fiber 0g	**0%**
Sugars 5g	
Protein 5g	
Vitamin A	**4%**
Vitamin C	**2%**
Calcium	**20%**
Iron	**4%**

4-1. WHAT ARE YOU EATING?

The government has created guidelines for how much of various vitamins, minerals, and other substances a person should eat or drink each day to be healthy. These guidelines are then used to create labels (like the one shown above) to inform the public about the nutritional content of food.

According to the sample label above, one serving of Cheesy Mac macaroni and cheese contains 20% of the recommended daily amount of calcium. Doctors recommend a person have a total of 1300 mg calcium each day. How many milligrams of calcium are in one serving of Cheesy Mac?

Your task: With your team, determine how many milligrams of calcium are in one serving of Cheesy Mac macaroni and cheese. Look for more than one way to solve the problem, and be ready to explain all of your **reasoning**.

Discussion Points

What information do we have about the part?

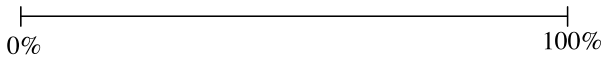
What do we know about the whole?

How could we represent this situation with a number line?

Further Guidance

4-2. To help you represent the situation in problem 4-1, copy the number line below on your paper.

Recommended Daily Intake of Calcium



a. With your team, decide how to **partition** the line (divide it into equal parts) so that 20% is shown. Why did you choose to make that number of parts? Is there another way that you could have divided the line?

b. The recommended daily intake of calcium is 1300 milligrams. Where should 1300 milligrams be labeled on the number line? Add this number to your diagram and **justify** your decision.

c. Use your diagram to help you decide how much calcium is in one serving of Cheesy Mac. Record your thinking.

Further Guidance section ends here.

4-3. One way to write a percentage is as a **ratio** (comparison) of parts out of 100. For example, the ratio $\frac{20}{100}$ represents 20 parts out of 100 total parts.

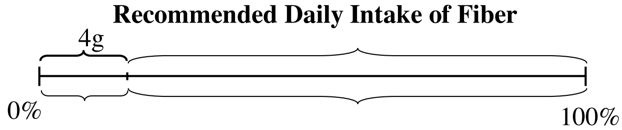
a. Jill represented the amount of calcium in one serving of Cheesy Mac with the ratio $\frac{260}{1300}$.

- What does the 260 represent?
- What does the 1300 represent?

b. The ratios $\frac{20}{100}$ and $\frac{260}{1300}$ are two different ways to compare the calcium in one serving to the recommended daily amount. How can you show that the ratios are equivalent (the same)?

4-4. One granola bar contains 4 g of dietary fiber. The label says that 4 g is 16% of the daily recommended amount. Louis decided to draw a diagram like the one below to understand this situation.

Recommended Daily Intake of Fiber



a. Copy the diagram on your paper and add the label for 16%.

b. How many grams of fiber are recommended each day? How can you show this with equal ratios?

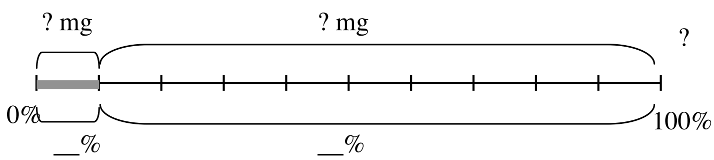
c. What percent of fiber should Louis get from other foods? Why is this percent equivalent to the ratio $\frac{21 \text{ g}}{25 \text{ g}}$?

d. What other amounts are missing on the diagram? Add labels for all parts, percents, and the whole.

e. Chris is eating cookies that contain 12 g of dietary fiber, which he says is 48% of the recommended daily amount. How can you use ratios to check that 12 g is equivalent to 48%?

4-5. One large carrot contains approximately 6 mg of Vitamin C. The recommended daily intake of Vitamin C is 60 mg. Resa wanted to find out what percentage of her daily Vitamin C she gets from one carrot. She started with a line divided into 10 parts.

Recommended Daily Intake of Vitamin C



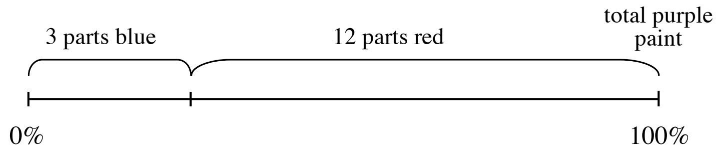
a. Why do you think she divided the line segment into 10 parts?

b. Copy the diagram on your paper and fill in the missing labels.

c. The ratio $\frac{6 \text{ mg}}{60 \text{ mg}}$ represents the portion of Vitamin C in one large carrot. Work with your team to find this ratio in the diagram. Where do you see each amount? What other ratio could you write that would be equal to this?

d. Use the diagram to help you find and write at least two more ratios on the number line that are equal to each other.

4-6. Resa was mixing blue and red paint to create purple paint. She created the drawing below to show the portions of blue paint to red that she used.



a. What does the picture tell you about the paint mixture? What statements can you make?

b. If you have not stated it yet, what percent of the paint is blue? What percent of the paint is red? **Justify** your answer.

4-7. **Additional Challenge:** Turner Middle School has 110 boys. Fifty-six percent of the students in the school are girls. How many students go to this school?

a. Create a model like Resa's from problem 4-5. Label the percentage of girls, the percentage of boys, and the number of boys on your drawing, as well as 0% and 100%.

b. How many students go to the school? How do you know?

c. How many girls go to the school? Explain your **reasoning**.

4-8. **Additional Challenge:** Regina is making paper flowers as decorations for the fall dance. She has made 40 flowers so far, and she is 16% finished. If she plans to finish making 70% of the flowers tonight, how many more will she need to make? Show your work.

MATH NOTES

METHODS AND MEANINGS

Equivalent Ratios

A **ratio** is a comparison of two amounts. A ratio can be written in words, as a fraction, or with colon notation. Most often in this course we will write ratios as fractions or state them in words.

For example, if there are 28 students in a math class and 15 of them are girls, we can write the ratio of the number of girls to the number of students in the class as:

15 girls to 28 students $\qquad \frac{15\text{ girls}}{28\text{ students}} \qquad$ 15 girls : 28 students

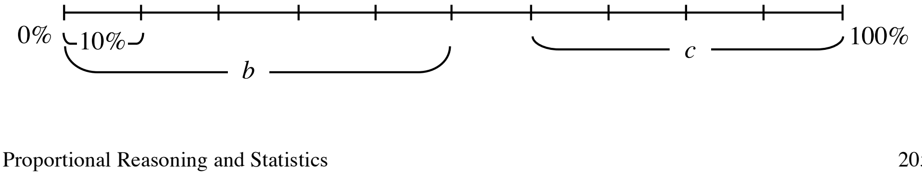
You used a Giant One to write equivalent fractions in Chapter 1. To rewrite any ratio as an **equivalent ratio**, write it as a fraction and multiply it by a fraction equal to one. For example, you can show that the ratio of girls to students is the same for a larger classroom using a Giant One like this:

$$\frac{5\text{ girls}}{9\text{ students}} \cdot \frac{3}{3} = \frac{15\text{ girls}}{27\text{ students}}$$

Equivalent fractions (or ratios) can be thought of as families of fractions. There are an infinite number of fractions that are equivalent to a given fraction. You may want to review the basis for using a Giant One — the Multiplicative Identity — in the Math Note in Lesson 1.2.3.

Review & Preview

4-9. Find the missing values on the diagram below. Assume that the line is evenly divided.



4-10. A fish tank that holds 80 gallons of water is 55% full.

a. Create a drawing like Louis' in problem 4-4 to represent this situation.

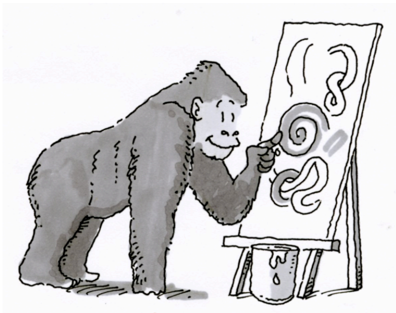
b. How many gallons are in the tank now? How many more gallons are needed to fill the tank?

4-11. Fill in the missing numbers in each number sentence.

a. $\frac{5}{8}\cdot\frac{3}{3}=\frac{?}{?}$ b. $\frac{9}{15}\cdot\frac{?}{?}=\frac{?}{60}$ c. $\frac{7}{20}\cdot\frac{?}{?}=\frac{?}{110}$ d. $\frac{44}{100}\cdot\frac{?}{?}=\frac{?}{60}$

e. What **strategies** did you use to find the numbers in the Giant One in parts (b), (c), and (d)?

4-12. Okie is a western lowland gorilla that lives at the Franklin Park Zoo near Boston, MA. He loves to finger paint and many of his paintings have been sold because their colors are so interesting. One painting was sold for five times the amount of a second, and a third was sold for $1500. If the total sale was for $13,500, how much did the most expensive painting sell for?



Set up a 5-D Process table and solve the problem. Then represent the relationships using a variable.

4-13. Simplify the following expressions.

a. $\frac{2}{3}+\frac{4}{5}$ b. $\left(\frac{4}{5}\right)\left(\frac{2}{3}\right)$ c. $\frac{4}{5}-\left(-\frac{2}{3}\right)$

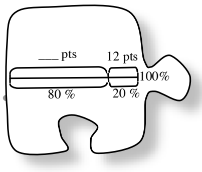
4-14. Lucy keeps track of how long it takes her to do the newspaper crossword puzzle each day. Her recent times (in minutes) were:

8 22 19 12 18 19 10 35 12 19 16 21

a. What is the median of her data?

b. What is the mode?

4.1.2 What is the percentage of the whole?



Finding and Using Percentages

How are sales advertised in different stores? In a clothing store, items are often marked with signs saying, "20% off" or "40% discount." In a grocery store, sale items are usually listed by price. For example, pasta is marked, "Sale price 50¢," or boxes of cereal are marked "$2.33 each." In the clothing store you are able to see how much the discount is, but the price you will pay is often not stated. On the other hand, sometimes in the grocery story it is not possible to tell the size of the discount. It might only be a small fraction of the original price.

The actual dollar amount of the discount and the percentage comparing it to the whole are important information that help you decide if you are getting a good deal. Today you will create complete information about a sale situation from the information given in a problem.

By the end of this lesson, you will be expected to answer the following target questions:

How can I find a percentage of a whole?

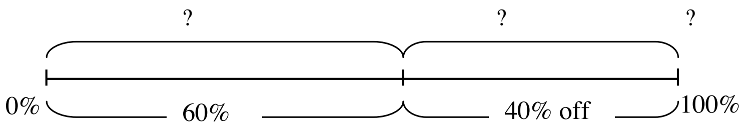
How can I find a percent if you have two parts that make a whole?

How can I find the whole amount if you know the parts?

4-15. Marisa is always looking for a great deal while shopping. She found a sale rack where all of the jeans are marked 40% off. Her favorite jeans regularly cost $65.

a. To figure out if she has enough money to buy a pair ofjeans, Marisa decides to estimate. She thinks that the jeans will cost approximately $30. Is her estimate reasonable? Explain your thinking.

b. To find the exact answer, Marisa created the diagram below. How could she add marks to evenly partition the line? Partition the line and calculate the missing values.



Problem continues on next page. →

4-15. *Problem continued from previous page.*

c. How much money will Marisa save? What is the price she will have to pay?

d. Marisa wants to check her answer from part (c). How could she use the amount she saved and the original price to verify that she received a 40% discount?

4-16. At the same sale, Kirstin sees a shirt that originally cost $50 on sale for just $37.50.

a. Estimate the percentage of the discount on the shirt.

b. Draw a diagram to represent this situation. Label all the parts.

c. What percent is the discount on Kirstin's shirt?

d. What is the relationship between the discount, sale price, and the original price? Write a statement that shows the relationship between the sale price, discount, and original price.

4-17. So far in this section, you have used a linear model to represent percent problems in various contexts.

a. Obtain a Lesson 4.1.2 Resource Page (available at www.cpm.org/students) and use the linear models to find parts and wholes when different types of information are provided on the diagrams.

b. For each diagram, write at least three statements describing how parts and percentages are related. Some statements are started for you.

4-18. When Kirstin was about to pay for her clothes, she realized that she had forgotten to include an 8% sales tax.

a. The belt Kirstin wants to buy costs $15 and she does not have paper to draw a linear model. To estimate tax, she figured that calculating 10% would be close enough. Explain how Kirstin might find 10% of $15 without a linear model.

b. Calculate exactly how much the 8% sales tax will cost Kirstin. What will be the total amount she will have to pay to the store?

MATH NOTES

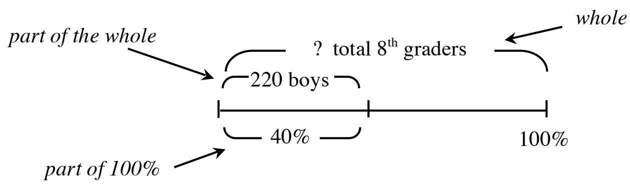
METHODS AND MEANINGS

Part to Whole Relationships

Percentages, fractions, and decimals are all different ways to represent a portion of a whole or a number. Portion-whole relationships can also be described in words.

We can represent a part to whole relationship with a linear model like the one below. To solve a percentage problem described in words, you must first identify three important quantities: the percent, the whole, and the part of the whole. One of the quantities will be unknown. A diagram can help to organize the information. For example:

> There are 220 boys in the 8^{th} grade class. If boys make up 40% of the 8^{th} graders, how many students are in the whole class?



Once the parts have been identified, you can use **reasoning** to extend the part to the whole. For example, if 220 students are 40% of eighth graders, then 10% must be $220 \div 4 = 55$. Then 100% must be $55 \cdot 10 = 550$ students. Another way to solve the problem is to find the ratio of 220 boys to the whole (all students) and compare that ratio to 40% and 100%. This could be written

$$\frac{40}{100} \cdot \frac{}{} = \frac{220}{?}\text{, then } \frac{40}{100} \cdot \frac{5.5}{5.5} = \frac{220}{?}$$

so the total number of 8^{th} graders is 550.

To remember how to rewrite decimals or fractions as percents and percents as fractions or decimals, refer to the Math Notes box at the end of Lesson 1.3.1.

Review & Preview

4-19. A shade of orange paint is made with 5 parts red paint and 15 parts yellow paint.

a. What percent of the paint is red?

b. What is the simplified ratio of yellow to red paint?

4-20. Janelle earned 90% on a test and got 63 points. How many total points were possible on the test? Draw a diagram to organize your information before solving the problem.

4-21. Use graph paper to solve the following problem.

a. Draw a four-quadrant graph and label each axis. Plot the following ordered pairs: $(-3,2)$, $(-8,2)$, $(-10,8)$, $(-5,8)$. Connect the points in the order given as you plot them, then connect the fourth point to the first one.

b. Describe the shape on your graph. What is its area?

4-22. Copy and simplify each expression.

a. $7+(-3)$ b. $(10)(-5)$ c. $-5+6$ d. $(-2)\div(-2)$

4-23. When solving a problem about the perimeter of a rectangle using the 5-D Process, Herman built the expression below.

$$\text{Perimeter} = x + x + 4x + 4x \text{ feet}$$

a. Draw a rectangle and label its sides based on Herman's expression.

b. What is the relationship of the base and height of Herman's rectangle? How can you tell?

c. If the perimeter of the rectangle is 60 feet, how long are the base and height of Herman's rectangle? Show how you know.